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Tests of peak flow scaling in simulated self-similar river networks

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Abstract

The effect of linear flow routing incorporating attenuation and network topology on peak flow scaling exponent is investigated for an instantaneously applied uniform runoff on simulated deterministic and random self-similar channel networks. The flow routing is modelled by a linear mass conservation equation for a discrete set of channel links connected in parallel and series, and having the same topology as the channel network. A quasi-analytical solution for the unit hydrograph is obtained in terms of recursion relations. The analysis of this solution shows that the peak flow has an asymptotically scaling dependence on the drainage area for deterministic Mandelbrot–Vicsek (MV) and Peano networks, as well as for a subclass of random self-similar channel networks. However, the scaling exponent is shown to be different from that predicted by the scaling properties of the maxima of the width functions.

Keywords: Self-similar networks; Peak flow; Scaling

1. Introduction

Empirically observed scaling or power-law relations between regional flood quantiles and drainage areas [3,20] are the result of complex interactions among numerous physical processes governing runoff generation and transport. These include the generation of surface and subsurface runoff involving precipitation, infiltration, and evapotranspiration, and the transport of water across hills and through a river network. A physical understanding of regional statistical scaling relationships in peak flows, and predictions from ungauged basins, is a long-standing open problem. Recent progress towards solving this problem is based on applications and development of new theories and models that involve notions of self-similarity and scale invariance [6]. Building on this recent work, we investigate how the transport of water obeying mass conservation, and attenuation due to storage in channel networks, combine with their self-similar topologic or branching structure in determining the spatial scaling structure of peak flows.

Our focus here is on routing rather than on the spatial and temporal variability of precipitation and runoff generation; see Gupta et al. [5], Menabde and Sivapalan [13], and Troutman and Over [24] for an investigation concerning the effect of precipitation on scaling exponents in idealised deterministic self-similar networks. For the present purpose, we simplify the runoff input into a network to be spatially uniform and instantaneous, and the flow is routed by a link-based mass conservation equation [6]. A time-varying solution of this equation requires that a physical relationship be known between storage and discharge for each link in a network. To better understand the analytical structure of this complex equation, we ignore the empirically observed downstream hydraulic-geometric variations in velocity [7,9], which makes this relationship linear. Therefore, one can formally view this formulation as a network of linear reservoirs in the sense of Nash [14], which are topologically connected in series and parallel according to the topology of a channel network.

The proposed approach leads to results different from those based on the analysis of scaling properties of the width function maxima assuming a constant velocity approximation [5,27]. The latter assumes that every drop of water in the channel network travels to the

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outlet with the same velocity v, and the flow does not attenuate. It is well known that within this approximation the instantaneous unit hydrograph (IUH) at the outlet of a catchment is given by

$$G(t) \propto W(vt), \tag{1}$$

where $W(x)$, $x > 0$ is known as the width function [4,8,17]. It is defined as the total number of channel links at a distance x from the outlet. It follows from the definition that $W(x)$ is a step-wise constant function taking positive integer values. If all links have the same drainage area then the scaling exponent of peak flows for the IUH is the same as the scaling exponent for the maxima of the width function. This approach has been used by Gupta et al. [5] and Gupta and Waymire [6] to estimate the spatial scaling exponent of peak flows in a Peano network, and by Veitzer and Gupta [27] in random self-similar river networks. In this paper, we show that storage due to flow routing in a channel network produces a power-law relationship in an asymptotic sense in the limit of large drainage area. But it modifies the flood scaling exponent as given by the width function maxima in a significant way.

In order to solve the mass conservation equation on a network, it is necessary to specify the network topology. We consider classes of channel networks with self-similar topologies. A notion of mean self-similarity in network topology was first proposed by Tokunaga [22,23]. Let T_v denote the number of lateral streams of order ω joining a stream of order $\omega + v$. A network is defined as "mean self-similar" if the coefficients T_v do not depend on ω. In principle, any given set of T_v defines a self-similar network. Tokunaga [23] considered a subclass of this model with a parameterisation

$$T_v = ac^{v-1}, v \geqslant 1, \tag{2}$$

which is defined as Tokunaga self-similarity [26]. The number of streams of different order ω is governed by a recursion relation [21,23],

$$N_\omega = 2N_{\omega+1} + \sum_{v=1}^{\Omega-\omega} T_v N_{\omega+v}, \tag{3}$$

where Ω is the maximum stream order in the network. This equation can be solved to show that the classic Horton law of stream numbers holds in the limit of large order. It is expressed as,

$$N_\omega / N_{\omega-1} \to R_B, \quad \omega \to \infty. \tag{4}$$

An expression for Horton's bifurcation ratio R_B is given by,

$$R_B = \frac{(2+a+c) + \sqrt{(2+a+c)^2 - 8c}}{2}. \tag{5}$$

Recent analysis of a dozen or so large basins [15] showed that the empirically observed bifurcation ratios are typically in the range between 4.1 and 4.7, which can be predicted by Eq. (5). This provides strong empirical evidence that the Tokunaga model is a better representation of real catchments than the well known random model [18,19], which predicts that $R_B = 4$. Tokunaga [23] showed that the random model exhibits self-similarity with generator parameters in (2) given by $a = 1$ and $c = 2$. A Tokunaga network with these parameter values is called the "average Shreve model".

A major limitation of Tokunaga theory however is that it only models averages, and not the statistical variability that is so prevalent in the real networks. A random self-similar network (RSN) theory has been developed by Veitzer and Gupta [26] to address this serious limitation of the Tokunaga theory. Unlike the Tokunaga theory, the RSN model is statistical in nature. Like the Tokunaga theory, it predicts the Horton law of stream number holds asymptotically as order $\omega \to \infty$, and that R_B can take values over a wide range. In fact, it has been shown that a subclass of RSN model exhibits Tokunaga self-similarity given in (2) in the limit of large order.

A reformulation of Horton laws in terms of probability distributions involving concepts of statistical self-similarity has been recently proposed by Peckham and Gupta [16]. It represents a new form of statistical order connecting a given variable at different spatial scales in terms of probability distributions. The classical Horton laws in terms of means represent special case of these "generalised Horton laws". For a random variable X, a generalised Horton law can be expressed as,

$$X_\omega \stackrel{d}{=} R_X X_{\omega-1}. \tag{6}$$

The new RSN theory predicts that Eq. (6) holds for geometric and topologic variables asymptotically as order $\omega \to \infty$. Comparisons of theoretical predictions with data also suggest that asymptotic convergence is fast, which makes it very useful for applications. Another way to generalise Tokunaga theory was recently proposed by Cui et al. [2], who assumed that the channel numbers are random realisations from a negative binomial distribution with a mean defined by the Tokunaga parameters a and c.

In this paper, we solve the mass conservation equation on a self-similar network, which gives us a quasi-analytical solution for the IUH in the form of a recursion relationship. This equation is solved numerically to compute hydrographs at multiple spatial locations in a network as a basis for investigating spatial scaling properties of peak flows. This quasi-analytical solution is described in Section 2. In Section 3 we apply this routing to deterministic self-similar networks, and in Section 4 to RSNs, and investigate how it modifies the spatial scaling of peak flows given by the maxima of the width functions. The key conclusions and problems for future research are briefly discussed in Section 5.

2. Flow routing on channel networks

We assume that all links in a channel network have the same length and the same local drainage area (this condition holds for real catchments on the average). Each link is indexed by k, which denotes the total number of upstream links draining into it. In terms of network magnitude m denoting the total number of source streams, it is well known that $k = 2m - 1$, which is proportional to the total drainage area draining into this link. The mass conservation equation for every link in the network can be written in the form [6]

$$\frac{\mathrm{d}S(k,t)}{\mathrm{d}t} = Q_{\mathrm{in}}(k,t) - Q_{\mathrm{out}}(k,t), \quad k = 1,3,5,\ldots,\ t \geqslant 0, \tag{7}$$

where $Q_{\mathrm{out}}(k,t)$ is the discharge at the bottom of the link k, and $Q_{\mathrm{in}}(k,t)$ consists of three parts:

$$Q_{\mathrm{in}}(k,t) = Q_{\mathrm{out}}(k_1,t) + Q_{\mathrm{out}}(k_2,t) + R(k,t), \tag{8}$$

where $Q_{\mathrm{out}}(k_1,t)$ and $Q_{\mathrm{out}}(k_2,t)$ are the discharges from the two upstream links indexed by k_1 and k_2, respectively, and $R(k,t)$ is the runoff contribution from hillslopes to the channel storage. As we consider here only the response of a channel network to a spatially uniform rainfall, we neglect the runoff generation processes on hillslopes, and assume that the whole volume of rainfall falling on the hillslopes immediately gets into the channels.

The water balance equation must be supplemented by a storage–discharge relationship, to solve the system of equations iteratively over the entire network. Since a link is the smallest spatial unit in our theory, we ignore the sublink hydraulic variability, and assign a "spatial mean" property to each link. By definition, link discharge $Q_{\mathrm{out}} = WDv$, where, W is mean link width, D is mean link depth, and v is mean link velocity. Similarly, link storage is defined as, $S = DWL$, where L is mean link length. These two definitions produce a relationship between discharge and storage as,

$$Q_{\mathrm{out}}(k,t) = \frac{v}{L} S(k,t). \tag{9}$$

As already discussed in the introduction, velocity in a network varies as a function of discharge both in downstream direction and at-a-station [7], but the link to link variations in a channel network are random, and do not show any specific trends. We assume here that the mean link length L is approximately a constant over the entire network. Moreover, to understand the analytical structure of the solution of Eqs. (7)–(9), we further assume that v is a constant velocity parameter. This assumption makes Eq. (9) formally the same as the well known equation for a linear reservoir in the sense of Nash [14]. The assumption of an instantaneously applied rainfall is equivalent to the assumption that all links initially receive the same volume of water, S_0. This becomes an initial condition for the dynamics of flow in the network described by Eqs. (7)–(9). Therefore the term $R(k,t) = 0$, $t > 0$ in the dynamics.

Under above approximations, we are able to obtain an analytical representation for the IUH at the bottom of every link in a network. Substituting (9) into Eq. (7) and introducing a dimensionless time $t' = vt/L$, the solution of Eq. (7) can be written as a convolution. For the sake of notational convenience, from here onwards, we denote the dimensionless time by t instead of t'. Then the solution is

$$S(k,t) = \exp(-t)\left\{ S_0 + \int_0^t \exp(\tau)[S(k_1,\tau) + S(k_2,\tau)]\,\mathrm{d}\tau \right\}. \tag{10}$$

For external links the second term in (10) vanishes, and the solution has the form

$$S(1,t) = S_0 \exp(-t). \tag{11}$$

Eq. (10) can now be solved iteratively for every link in the network. For example, the solution for a link having two upstream external links and $k = 3$ has the form

$$S(3,t) = S_0 \exp(-t)(1 + 2t). \tag{12}$$

The reader can check that the general solution for an arbitrary link k has the form

$$S(k,t) = S_0 \exp(-t) \sum_{n=0}^{n_{\max}} C_n t^n. \tag{13}$$

The polynomial coefficients C_n can be found through the recursive relation

$$C_0 = 1, \quad C_n = \frac{C^1_{n-1} + C^2_{n-1}}{n}, \quad n \geqslant 1, \tag{14}$$

where C^1 and C^2 are polynomial coefficients for the two upstream links, and $n_{\max}$ is equal to the maximum number of consecutive upstream links for the given link k. An example application of Eqs. (13) and (14) is worked out for the Mandelbrot–Vicsek (MV) network in Section 3.

Eq. (13), in fact, represents a scaled version of an IUH for any link of a binary network. However, in order to obtain solutions of this equation, it is necessary to have specific information about the topology of a channel network for computing the polynomial coefficients in Eq. (14). Obviously, the topology of a channel network can be easily obtained from a map, or a digital elevation model (DEM) for a given basin, and Eqs. (7)–(9) can be solved numerically, if not analytically, using the recursive relation (14). This raises the question about the relevance of a theoretical understanding of network structure in modelling flows [1]. To appreciate this issue it is necessary to go beyond modelling a hydrograph at a fixed location, which has been a common practice in engineering hydrology. It was also the focus of the theories of geomorphologic unit hydrograph [17]. However, the signature of network topologic and geometric structure on flow hydrographs predicted by these theories was masked by flow attenuation due to runoff generation and channel storage, and it could not be clearly detected in data. By contrast, our focus here is on understanding the spatial variability of peak flows, and on the presence or absence of scaling relationships governing this variability. In this context, the importance of a theoretical understanding of network topology and geometry becomes a central piece of the puzzle. In the next two sections we study the scaling properties of peak flows using simulated self-similar networks and solving Eqs. (13) and (14) on these networks. Without the recent developments that have highlighted the presence of self-similarity in the topologic structure of real channel networks [26], it would be very difficult to understand peak flow scaling in a physically meaningful manner.

3. Deterministic self-similar networks

We consider here two examples of deterministic self-similar networks, namely, MV [11], and Peano [10,12] networks. The MV network is constructed as a self-similar binary tree, which is obtained by an iterative use of a tree-like generator shown in Fig. 1. As we consider here only topological networks the orientation of the generator is arbitrary. An example of the resulting network after two iterations is shown in Fig. 2. The total number of links after the nth step of iteration is 3^n, the Horton–Strahler order is $n+1$, and the IUH at the outlet is given by (13), where the order of the polynomial is $n_{\max} = 2^n - 1$. The maximum distance from the outlet (in terms of number of links) is 2^n, and the number of external links at this distance is also 2^n. The total drainage area A is proportional to the total number of links, i.e.,

$$A = a_0 3^n, \tag{15}$$

where a_0 is the local drainage area of every link. This leads to the following scaling dependence of the maximum of width function $W_{\max}$ on the drainage area A

$$W_{\max} = 2^n = 2^{\log_3 A/a_0} \propto A^{\alpha}, \quad \alpha = \log_3 2 \approx 0.63. \tag{16}$$

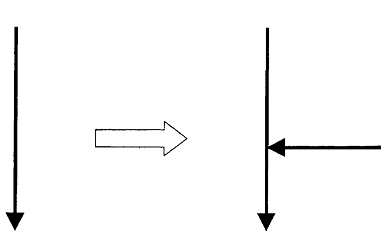

Fig. 1. The generator of MV network.

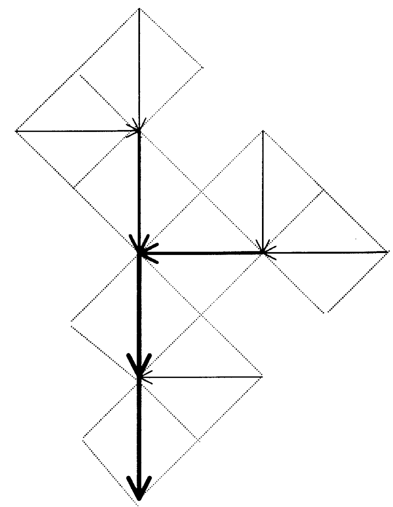

Fig. 2. MV network of order 3.

In order to study the scaling properties of peak flows we need to find the maximum of IUH as a function of drainage area. It is neither obvious that the IUHs given by (13) will have the same scaling properties as the width function nor is it obvious that the peak flows will scale at all. For example, the IUHs for three subcatchments of a fourth-order catchment are given by:

$$S(3) = \exp(-t)(1 + 2t), \tag{17}$$

$$S(9) = \exp(-t)\left(1 + 2t + t^2 + \frac{2}{3}t^3\right), \tag{18}$$

$$S(27) = \exp(-t)\left(1 + 2t + t^2 + \frac{2}{3}t^3 + \frac{1}{12}t^4 + \frac{1}{30}t^5 + \frac{1}{180}t^6 + \frac{1}{630}t^7\right). \tag{19}$$

The IUHs given by (17)–(19) are indexed by the total number of links, or their drainage areas, $S_0 = 1$, and t denotes dimensionless time. The maxima of these IUHs are: $S_{\max}(3) \approx 1.213$, $S_{\max}(9) \approx 1.910$, $S_{\max}(27) \approx 3.277$. For the external links we obviously have $S_{\max}(1) = 1$. If the peak flows were scaling it would be possible to approximate them by a single exponent β, i.e.,

$$S_{\max}(3^n)/S_{\max}(3^{n-1}) = 3^{\beta}. \tag{20}$$

However, the values of exponents for (17)–(19) are $\beta_1 = 0.176$, $\beta_2 = 0.413$, $\beta_3 = 0.476$, i.e., the IUHs' maxima are not scaling. One may conjecture that the exponents may rapidly converge to some limiting value, i.e., the peak flows may be asymptotically scaling. We were unable to show convergence analytically, or get an analytical estimate for the scaling exponent for a network of an arbitrary order. However, the maxima of peak flows could be easily analysed numerically. The results are shown in Fig. 3. As one can see, the peak flows are indeed asymptotically scaling with a scaling exponent $\beta = 0.49$. This value is substantially lower then

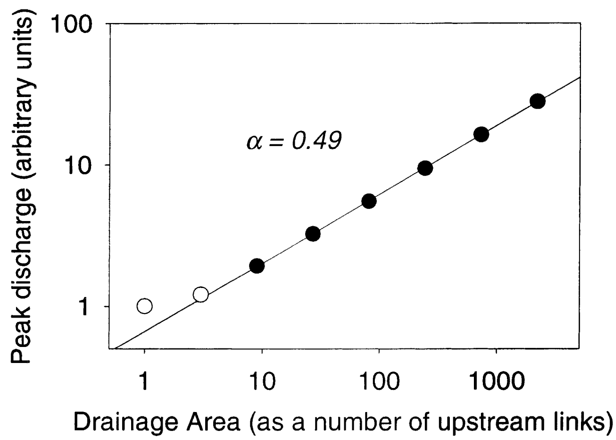

Fig. 3. Scaling of peak flow in the MV network.

the value 0.63 for the scaling exponent α of the width function, given by (16).

To investigate if the above feature is observed in other deterministic self-similar networks, we studied the scaling properties of peak flows in the Peano network. This is a deterministic self-similar ternary tree constructed as shown in Fig. 4. A geometrical consideration of the peak flow properties via the width function maxima leads to the scaling exponent value [5],

$$\alpha = \log_4 3 \approx 0.79. \quad (21)$$

However, the linear reservoir model leads to a different result based on Eqs. (10)–(14). Even though these equations were derived for binary networks, they can be easily generalised to consider ternary network, as is the case with Peano network. Repeating the same analysis as carried out above, we find that the peak flows in a Peano network are asymptotically scaling with the exponent $\beta \approx 63$ (see Fig. 5). In this case too, the predicted value of scaling exponent of peak flows is substantially lower then the one predicted by purely geometrical consideration.

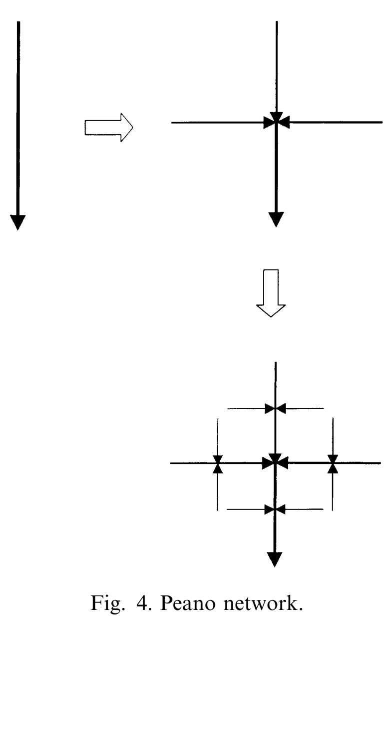

Fig. 4. Peano network.

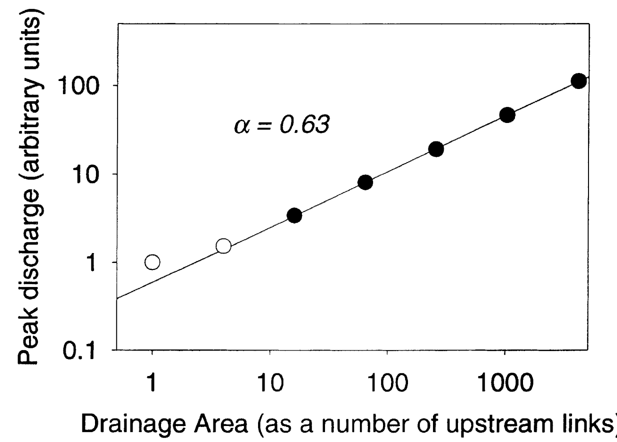

Fig. 5. Scaling of peak flow in the Peano network.

Both constructions, the MV and Peano networks, are deterministic and highly idealised. In the next section we consider the analysis of peak flow scaling for a subclass of RSNs which have topological properties similar to real channel networks.

4. Random self-similar channel networks

The RSN model of channel networks was introduced by Veitzer and Gupta [26]. The simulation of RSN proceeds in the same manner as the topological construction of the MV and the Peano networks, except that the replacement generators are suitably randomised. At each stage of the construction, each link in an existing network is replaced by a binary tree-like generator, which are chosen from a Bernoulli distribution of possible generators according to a specified probability parameter. Interior and exterior links are replaced from separate collections of replacement generators. A complete description of the construction process for random self-similar networks is given in [25,26]. We consider here a subclass of networks, which is general enough to reproduce the empirically observed properties of real catchments. The networks are constructed by starting with a single link and by the recursive use of one of three types of replacement generators (Fig. 6), having 1, 2, or 3 nodes, or it can be left unchanged as the fourth possibility. On every step of the construction the interior links are replaced with a generator with n_1 nodes with probability p_i, and by a generator with n_2 nodes with probability $(1 - p_i)$, and the exterior links are replaced with a generator with n_3 nodes with probability p_e, and by a generator with n_4 nodes with probability $(1 - p_e)$. It has been shown by Veitzer and Gupta [26] that this construction leads to an

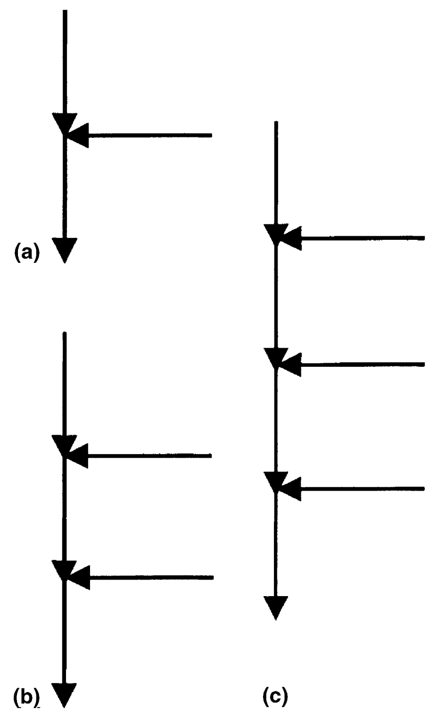

Fig. 6. Replacement generator in the RSN model.

asymptotically self-similar network, in the sense of Eq. (4), with the bifurcation ratio given by

$$R_B = \mu_i + \mu_e + 1, \tag{22}$$

where $\mu_i = n_1 p_i + n_2(1 - p_i)$ and $\mu_e = n_3 p_e + n_4(1 - p_e)$ are, respectively, the mean numbers of interior and exterior nodes in the replacement generator. The probabilities p_i and p_e are not related in general, but for simplicity we take them here to be the same. In this case the model still has enough degrees of freedom to produce a network with any prescribed value of the bifurcation ratio, R_B. In general, R_B does not define the RSN uniquely, and it can be seen from Eq. (22) that many different combinations of modelling parameters can generate the same R_B. We however, consider here R_B as the main topological characteristic of a network, and our aim is to study the dependence of scaling exponent of peak flows on the bifurcation ratio, R_B.

First, we consider the case when $p_i = p_e = 0.5$, $n_1 = 0$, $n_2 = 2$, $n_3 = 1$, and $n_4 = 3$. This construction leads to the average Shreve model [26]. The results of simulation for a typical realisation of an order eight network are shown in Figs. 7 and 8. These curves were obtained by analysing all the sub-basins of given order and calculating the mean value for each order. It can be seen that the peak flows as a function of the drainage area are asymptotically scaling with an exponent $\beta = 0.57$ and the width function is scaling with an exponent $\alpha = 0.58$.

The second set of parameters used in simulation was $n_1 = 1$, $n_2 = 2$, $n_3 = 2$, and $n_4 = 3$, with the generator probability p ($p_i = p_e = p$) varying in the range from

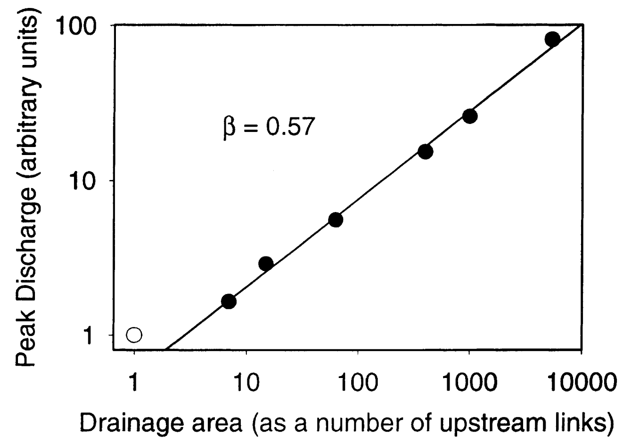

Fig. 7. Scaling of peak flow in the average Shreve model.

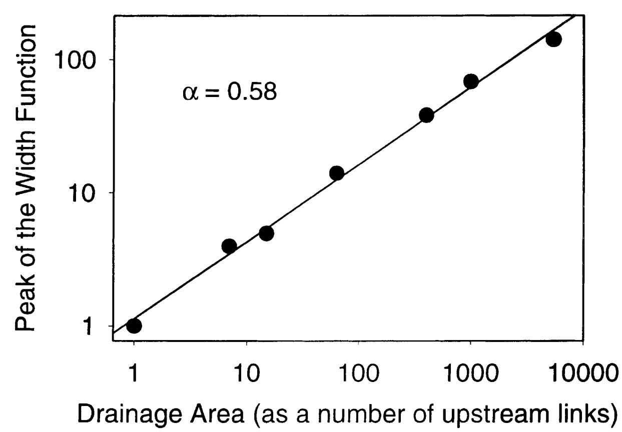

Fig. 8. Scaling of the peak of width function in the average Shreve model.

0.95 to 0.65. The corresponding bifurcation ratio, R_B, was, according to (22), in the range from 4.1 to 4.7, i.e., within the range, typically observed in the real catchments [15]. Although, the value for R_B given by (22) is only valid asymptotically [26] the convergence is quite fast to be useful in applications. In order to find out how well the theoretical value of R_B is reproduced in finite networks, we simulated a statistical ensemble of 100 networks of order 7 with the same set of parameters. The results of simulations for the particular case of $R_B = 4.2$ are shown in Fig. 9. It can be seen that while the bifurcation ratios for higher order streams are strongly biased and exhibit large fluctuations, the bifurcation ratios for the streams of the first three orders are very close to the theoretical one. In particular, the variance of R_B for the streams of order one and two is negligibly small. This fact makes it possible to use the theoretical value of R_B for the characterisation of individual simulated networks and to study the dependence of scaling characteristics of networks on bifurcation ratio.

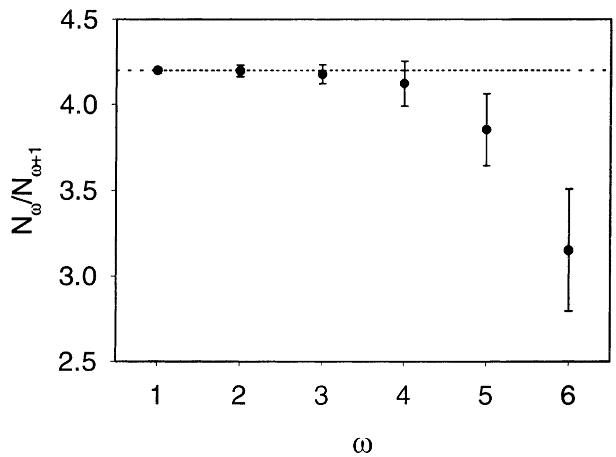

Fig. 9. Bifurcation ratios in the seventh-order RSN network ($R_B = 4.2$).

The dependencies of peak flows and width functions on drainage area for nested networks for one realisation of RSN with $R_B = 4.2$ are shown in Figs. 10 and 11. It can be seen that both the peaks of width function and the peak flows exhibit a good scaling behaviour, even for a single realisation. However, the scaling exponent for peak flows is not that much smaller than the exponent for the scaling of the width function maxima. This result is quite remarkable and non-intuitive given that the RSNs embody statistical variability in their topology. The same scaling behaviour is observed for simulated networks with bifurcation ratios in the range of $4.1 \leqslant R_B \leqslant 4.6$. For higher values of R_B the peak of width function still has a scaling dependence of the drainage area, whereas the peak flow cannot be approximated by a single straight line on a log–log scale (see Figs. 12 and 13).

Individual realisations of random networks with the same generator parameters show a high degree of variability in the scaling exponents. To study the statistical properties of the RSN model we simulated 100 realisations for each given R_B. The results are shown in Figs. 14 and 15. It can be seen that the scaling exponent of peak flow has a definite tendency to decrease with increasing bifurcation ratio and its

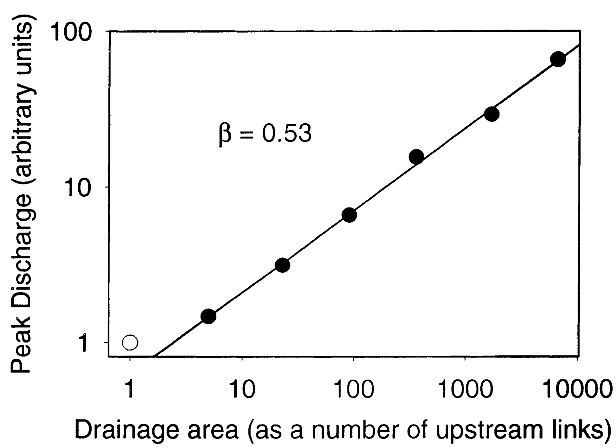

Fig. 10. Scaling of peak flow in the RSN model.

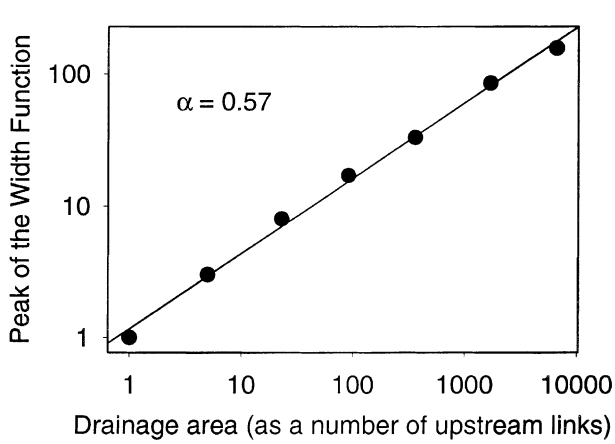

Fig. 11. Scaling of the peak of width function in the RSN model.

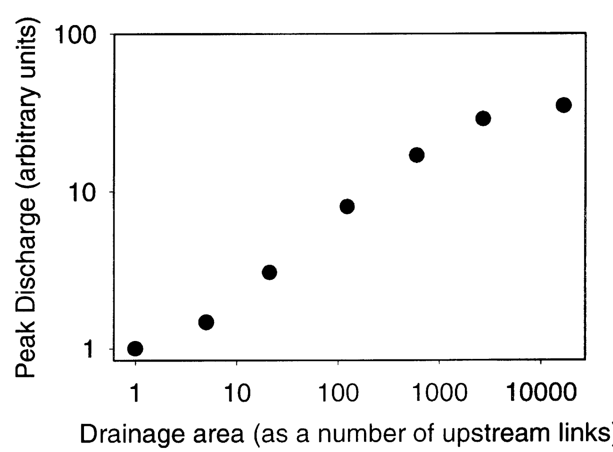

Fig. 12. Peak discharge in the RSN model.

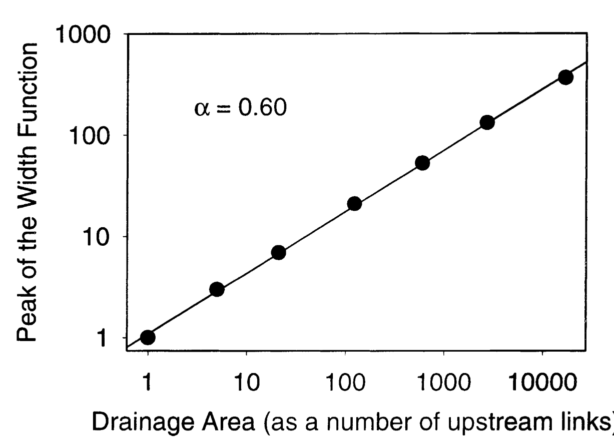

Fig. 13. Scaling of the peak of width function in the RSN model ($R_B = 4.7$).

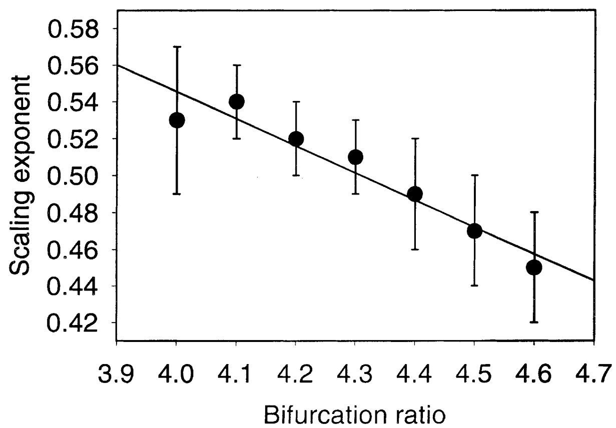

Fig. 14. Scaling exponent of peak flow as a function of bifurcation ratio.

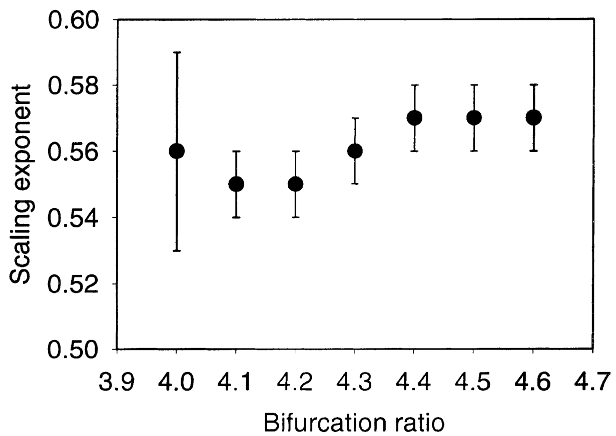

Fig. 15. Scaling exponent of width function as a function of bifurcation ratio.

dependence on R_B can be approximated by the linear function:

$$\beta = 1.131 - 0.146 R_B. \qquad (23)$$

The standard deviation is typically around 0.3. On the other hand, the scaling exponent for the peaks of width function does not have a strong dependence on R_B, and the standard deviation is substantially lower, typically about 0.1. The individual realisations of networks with the same parameters can exhibit large fluctuations. Say, for $R_B = 4.2$ the scaling exponent was in the range 0.46–0.55, and for $R_B = 4.6$ – in the range 0.39–0.53. Particularly large fluctuations were observed for the average Shreve model ($R_B = 4.0$). In this case the scaling exponent was in the range of 0.45–0.62. These large fluctuations mean that individual networks with very different bifurcation ratios can exhibit the same scaling properties when it comes to the peak flows. Since in reality we always have only one realisation of a network this means that the scaling exponent of the peak flow cannot be considered as a definitive attribute of the channel network topology. Of course, these conclusions are valid only for the linear storage–discharge approximation considered here. Incorporation of hydraulic-geometry gives rise to a nonlinear storage–discharge relationship. Some preliminary results on the scaling properties of peak flows in a network of nonlinear reservoirs are given elsewhere [13].

5. Conclusion

Recent attempts to understand the scaling properties of peak flows in terms of physical processes have considered highly idealised networks, such as the Peano basin, and assumed translational routing without attenuation [5,6]. In this paper, we have significantly generalised the scope of this investigation in two different ways. First, we considered a linear flow routing equation on a channel network, which explicitly included storage and flow attenuation. We studied the spatial scaling properties of peak flows for two kinds of deterministic self-similar networks, MV and Peano. It was shown that peak flow as a function of the drainage area exhibits asymptotically scaling behaviour in these two deterministic networks. Moreover, the scaling exponent for the peak flows is shown to be substantially lower then the value predicted by the maxima of the width functions.

The second generalisation involved considering RSNs that exhibit realistic topologies of natural channel networks [26]. For a subclass of RSN models a scaling behaviour of peak flows is observed for bifurcation ratios in the range of $4.0 \leqslant R_B \leqslant 4.6$. For larger values of R_B the peak flow dependence on the drainage area does not have a scaling behaviour, whereas the maxima of the width functions still scale. The ensemble average values of peak flow exponents show a definite tendency to decrease with the increasing bifurcation ratio. On the other hand, the scaling exponent values for individual realisations of random networks exhibit large fluctuations, so that networks with different bifurcation ratios may still have the same scaling exponent of peak flows. It should be noted that the scaling behaviour of peak flows based on a linear routing equation in a self-similar topological model of a channel network is by no means self-evident. The results in this work were obtained by combining an analytical and a numerical approach.

The results presented here should be extended in several directions. Let us mention a few important problems. In a new investigation [24], it has been observed that maxima of width functions for a subclass of RSNs obey generalised Horton law given by (6). This raises the question, if this result extends to peak flows generated through storage and flow attenuation as considered here. Second, the effect of nonlinear storage–discharge relations on peak flow scaling incorporating downstream and at-a-station hydraulic-geometric variations in flow velocity needs to be investigated [9]. Third, the effect of runoff generation processes on peak flow scaling needs to be investigated; see [13] for some preliminary results on the latter two problems. The above steps are necessary to develop a comprehensive hydrologic theory of floods. Such a theory will incorporate network/hillslope interactions in a physically realistic manner, and it will use concepts of scale invariance and statistical variability as its foundations. We wish to conclude with the observation that new, comprehensive sets of field measurements are badly needed to test this and other new theories as they develop.

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